Polygon-gluing Enumeration with Polynomial Bialgebra Partition

Matt Bernard[‡]
mb@ibmatt.org

‡ No Affiliation

Abstract We first prove arbitrary polygon-gluing line-breaking, spectral completion with respect to the standard irreducible representation $\mathbf{St}_n = \mathbb{C}^n/\mathbb{C}$ of $G = \mathsf{S}_n$ of dimension $n-1$ with conjugacy invariance $\langle f_1,\, f_2 \rangle = |G|^{-1} \sum_{g \in G} f_1(g) f_2(g)$ and ergodic decompositon $\bigoplus_{\pi \in \mathcal{R}} m(\pi)\, \pi$ of $V = \pi_0 \oplus \cdots \oplus \pi_{n-1}$ for $f : \mathsf{S}_n \mapsto \mathbb{C}$, where $\mathcal{R}$ is set of representatives of the isomorphism classes of irreducible G representations alongside non-negative integer multiplicities $m(\pi)$. We then prove the unique trace decomposition $\chi_V(g)\, \mathbf{tr}(g, V)$ for all given representations $\pi_r = (\mathbf{St}_n)$, $0 \leq r \leq n-1$, for every S_n, $\chi_V : \mathsf{S}_n \mapsto \mathbb{C}$; and we show the nontrivial application of the result in orbifold Euler-characteristic partition evaluation within modern context of a theory of genus-g moduli space $\mathcal{M}$ curves. In particular, we obtain the enumeration of $\pi : \mathcal{M} \to \mathcal{M} \otimes \mathcal{M}$ sequences of polynomials containing Catalan sequence, in terms of the (exponential) generating function $Q_\tau(k) = \frac{1}{(2m-1)!!} \sum_{0 \leq g \leq m/2} \varepsilon_g(m)\, k^{m+1-2g}$ for all π-free involution $\tau \in \mathsf{S}_{2m}$ or interchangeably mod-π distribution whose product over standard cyclic permutation σ has $m+1-2g$ cycles, such that $\varepsilon_g(m)$ is the number of ways to identify sides of a $(2m)$-gon in pairs with reverse orientation and, $(2m-1)!! = 1 \cdot 3 \cdot \cdots \cdot (2m-1)$, which is the number of ways to glue an oriented surface from the regular $(2m)$-gon, is precisely the cardinality of the conjugacy class of τ.

Keywords: Polynomial bialgebra, moduli space enumeration

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INTRODUCTION