
РУССКИЕ ФИРМЫ

老街音乐汇
РУССКИЕ ФИРМЫ ДРУЗЬЯ
中央大街1号
六桂福珠宝
六桂福珠宝


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SMOKING
哈
林志


Vladimir Manuilov
勿吸烟
SMOKING
志


哈工


NO SMOKING


请勿吸烟
NO SMOKING


GOOD
GIRL


曼哈顿大酒店


S·G·P·G
排水
承重
铸造厂制


排水
承重
铸造厂制


Cal
No Smoking


1920-24: The Lenz-Ising model
Lenz suggested the model to his student Ernst Ising, who proposed a specific form of interaction
Squares of two colors, representing spins s=±1
Nearby spins want to be the same, parameter x :
W(config)=x^#(+-neighbors)
Partition function
Z = ∑config W (config)
Probability P(cfg) =W(cfg)/Z
保持卫生

1924: Ernst Ising thesis
“no phase transition in dimension 1”
0 1 .... n n+1
+ – + + + – – + – – + – + + ?
Length n+1 chain, the leftmost spin is +
$Z = \Sigma_{conf.} x^{\#((+)(-)neighbors)} = (1+x)^n$
The rightmost spin is + for even powers of x
$Z_+ = \{ (1+x)^n + (1-x)^n \}/2$
So the probability
$P(\sigma(n)=+) = Z_+/Z = \frac{1}{2} + \frac{1}{2}\left(\frac{1-x}{1+x}\right)^n,$
which tends exponentially to ½
持卫生
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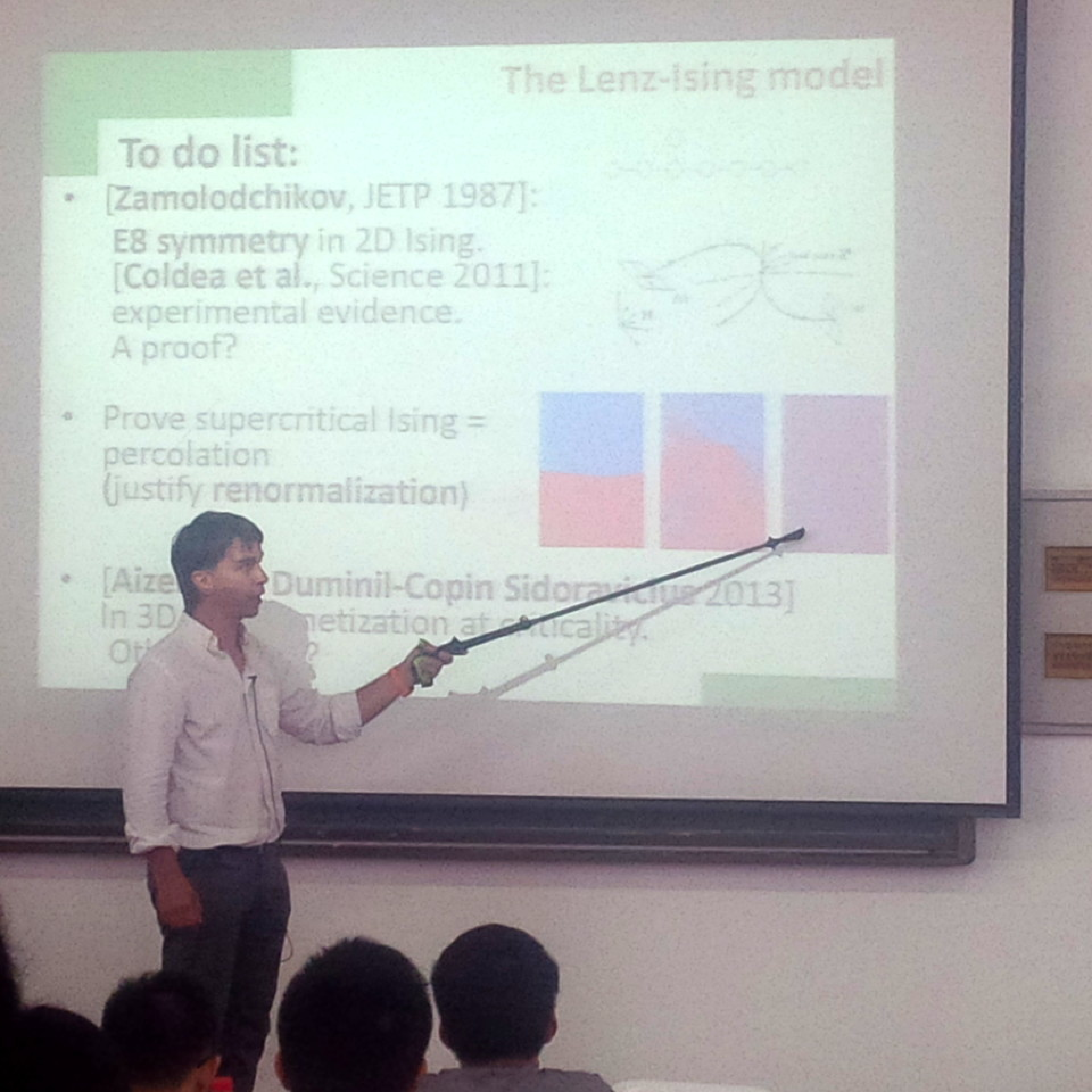
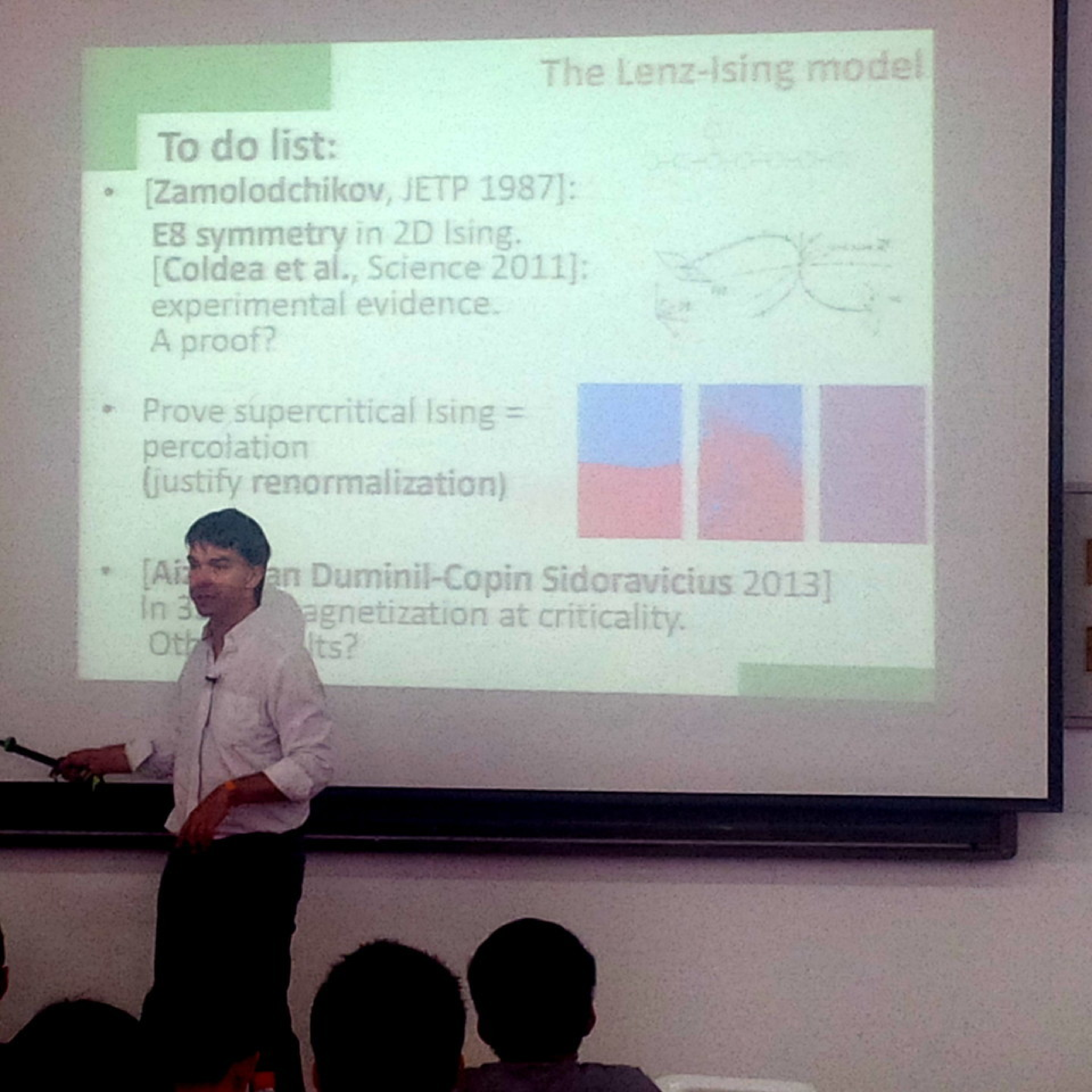
LITERATURE
SUM THE RELATION
OVER SOME TRIANGLE
INSIDE OK
ON THE BOUNDARY SOME EXTRA OR MISSING TERMS
= ERROR


The Lenz-Ising model
To do list:
[Zamolodchikov, JETP 1987]:
E8 symmetry in 2D Ising.
[Coldea et al., Science 2011]:
experimental evidence.
A proof?
Prove supercritical Ising =
percolation
(justify renormalization)
Duminil-Copin Sidoravicius 2013]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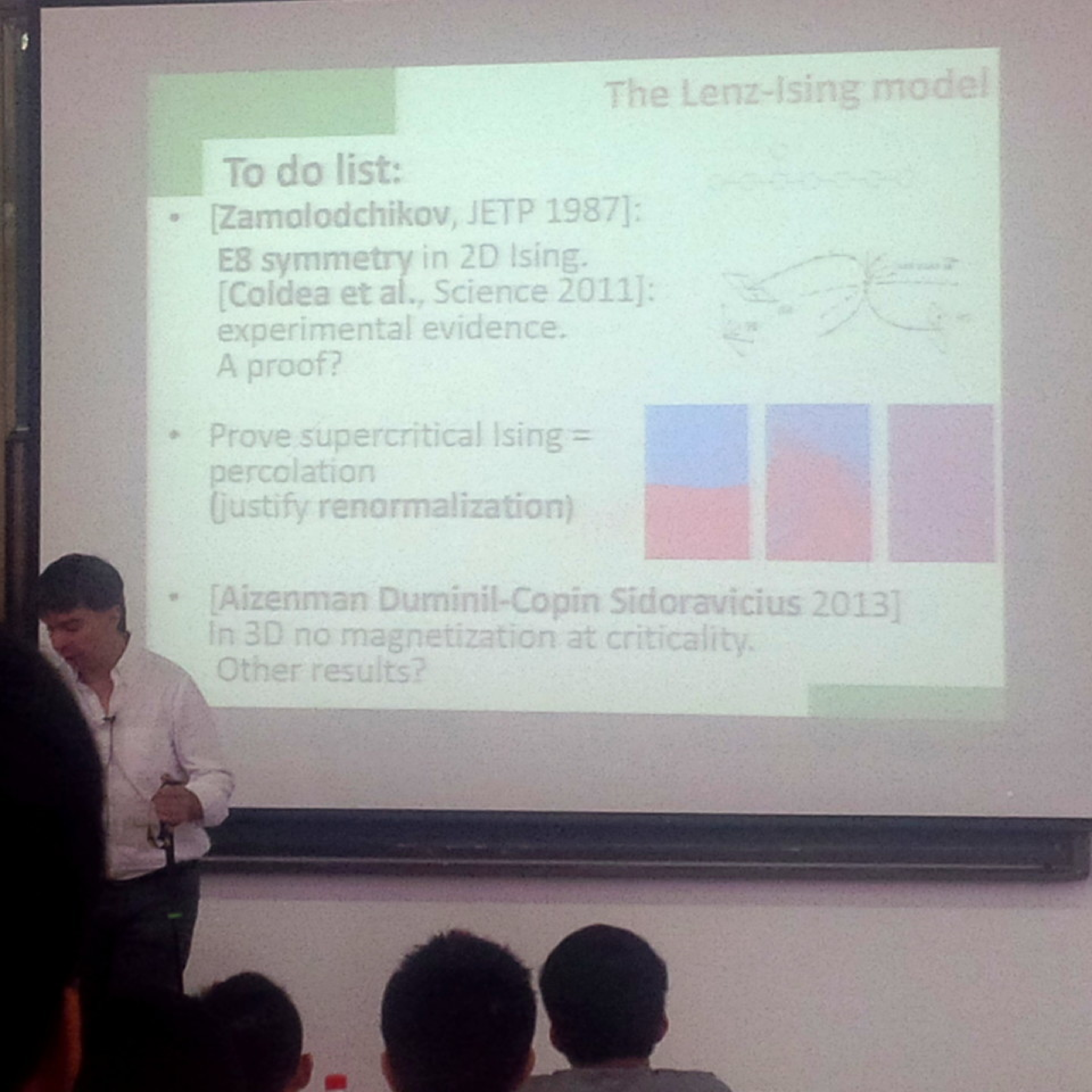
at criticality.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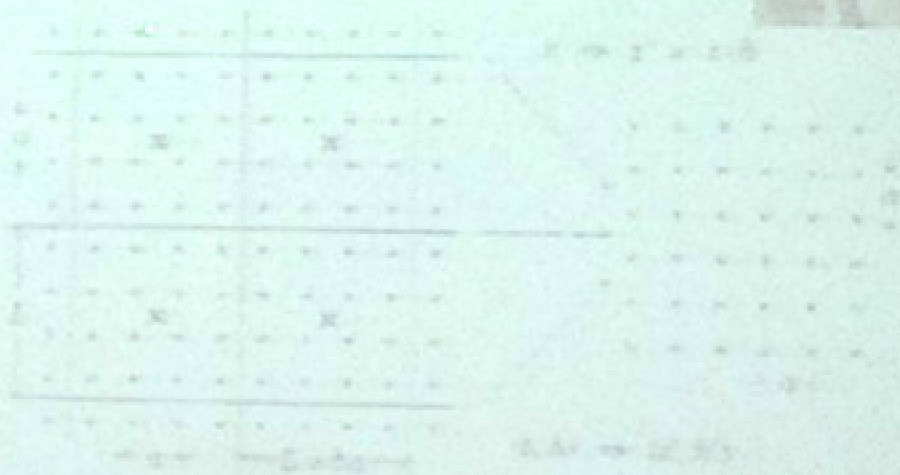
In 3D no magnetization at criticality.
Other results?

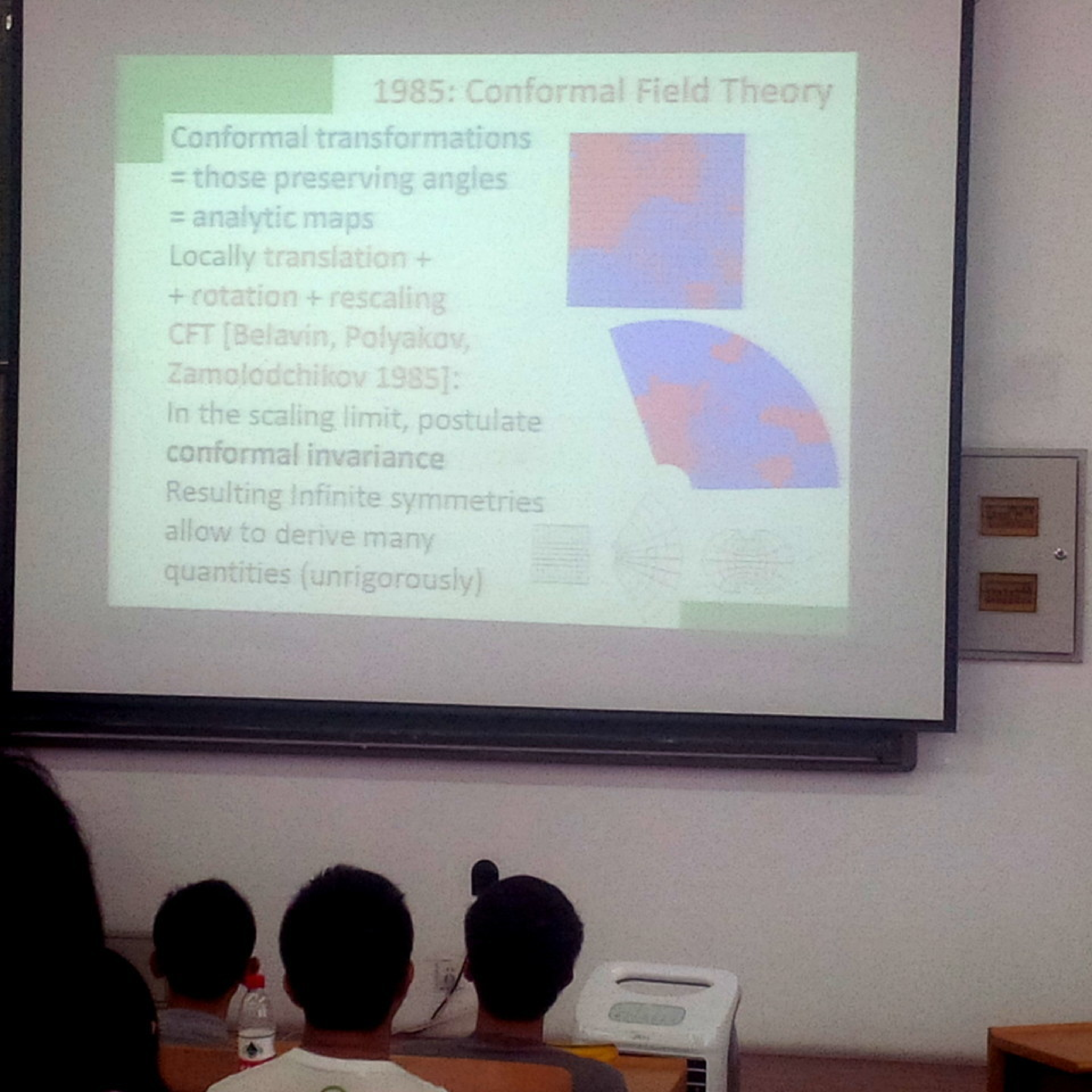
1951: Renormalization Group

Petermann-Stueckelberg 1951, ...
Kadanoff, Fisher, Wilson, 1963-1966, ...

Block-spin renormalization = rescaling + change of x



Conclusion:
At criticality the scaling limit is described by a "massless field theory"
The Curie critical point is universal and hence translation, scale and rotation invariant


1985: Conformal Field Theory
Conformal transformations
= those preserving angles
= analytic maps
Locally translation +
+ rotation + rescaling
CFT [Belavin, Polyakov,
Zamolodchikov 1985]:
In the scaling limit, postulate
conformal invariance
Resulting Infinite symmetries
allow to derive many
quantities (unrigorously)


瑞丽
你就是美人
YOU'RE


MAKHIN FOOD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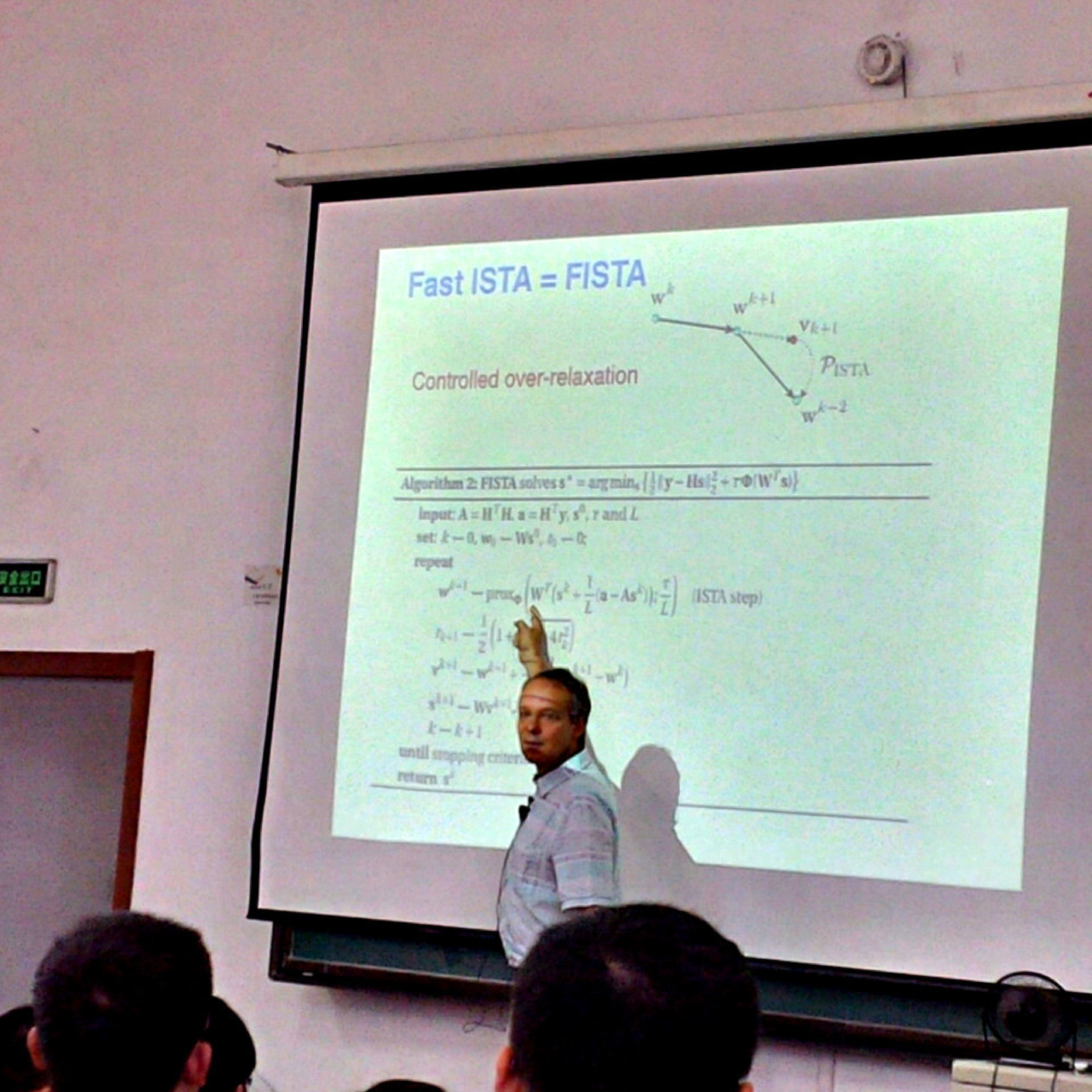
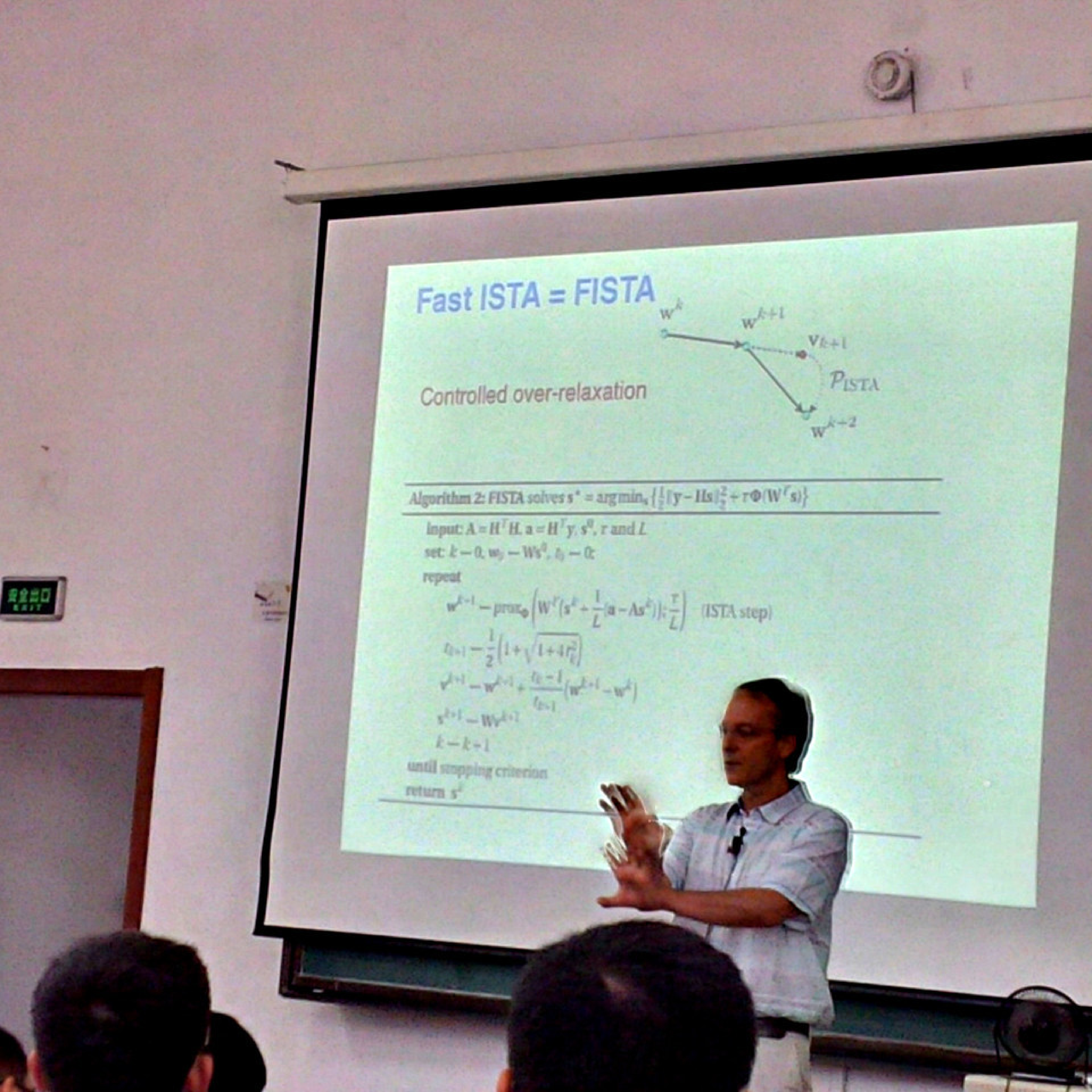
Hodge Theory
Harmonic 1-form
Each cohomologous class has a unique harmonic 1-form, which represents a vortex free, source-sink free flow field.
Theorem (Hodge)
All the harmonic 1-forms form a group, which is isomorphic to $H^1(M)$.
Theorem (Hodge Decomposition)
$\Omega^k(M) = Img\, d^{k-1} \oplus Img\, \delta^{k+1} \oplus H^k_\Delta(M)$.


Fast ISTA = FISTA
Controlled over-relaxation


Fast ISTA = FISTA
$\mathbf{w}^k$
$\mathbf{w}^{k+1}$
$\mathbf{v}_{k+1}$
$\mathcal{P}_{\mathrm{ISTA}}$
$\mathbf{w}^{k-2}$
Controlled over-relaxation
Algorithm 2: FISTA solves $\mathbf{s}^* = \arg\min_{\mathbf{s}} \left\{ \frac{1}{2}\|\mathbf{y} - \mathbf{H}\mathbf{s}\|_2^2 + \tau\Phi(\mathbf{W}^T\mathbf{s}) \right\}$
Input: $\mathbf{A} = \mathbf{H}^T\mathbf{H}$, $\mathbf{a} = \mathbf{H}^T\mathbf{y}$, $\mathbf{s}^0$, τ and L
set: $k \leftarrow 0$, $\mathbf{w}_0 \leftarrow \mathbf{W}\mathbf{s}^0$, $t_0 \leftarrow 0$;
repeat
(ISTA step)
until stopping criteri
return $\mathbf{s}^k$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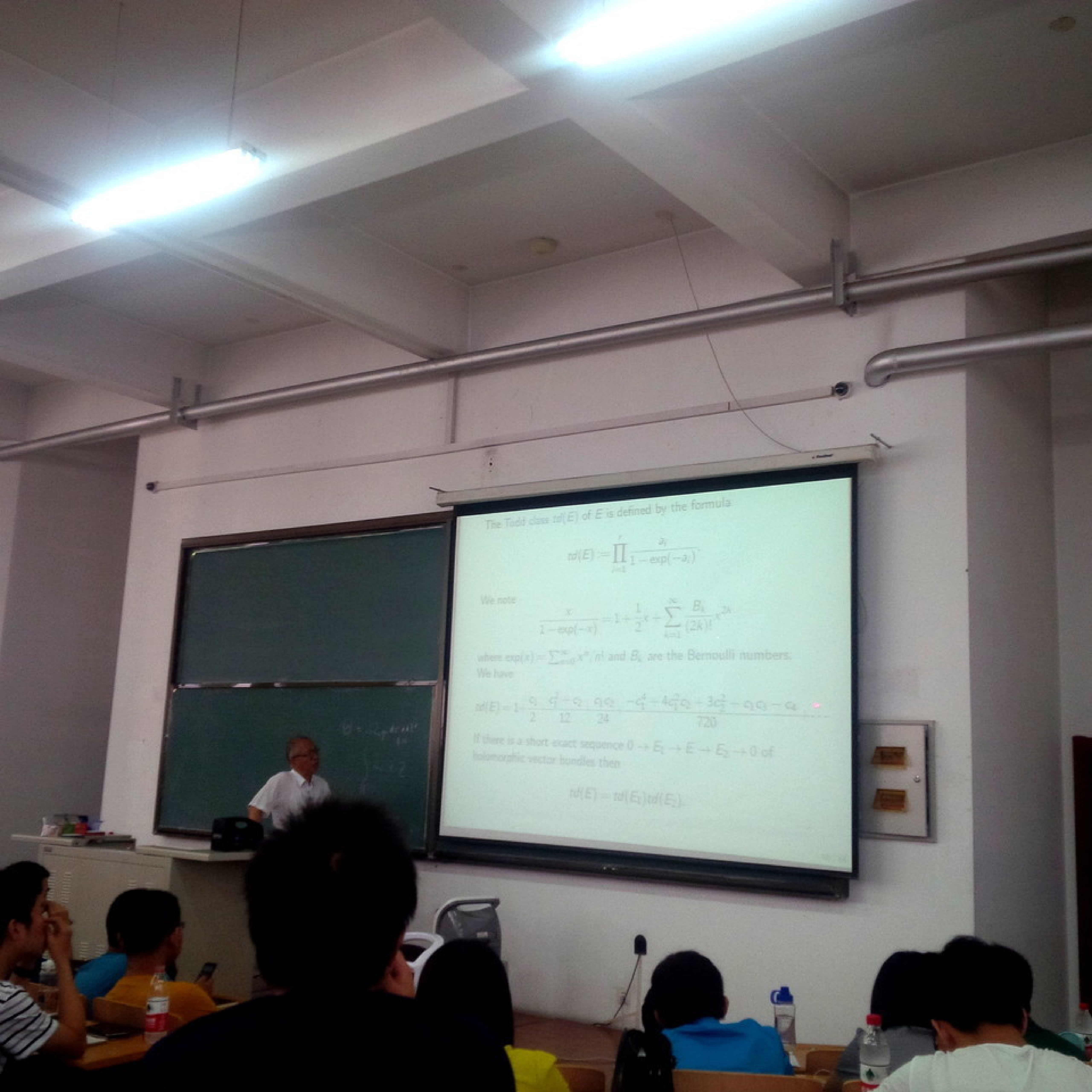
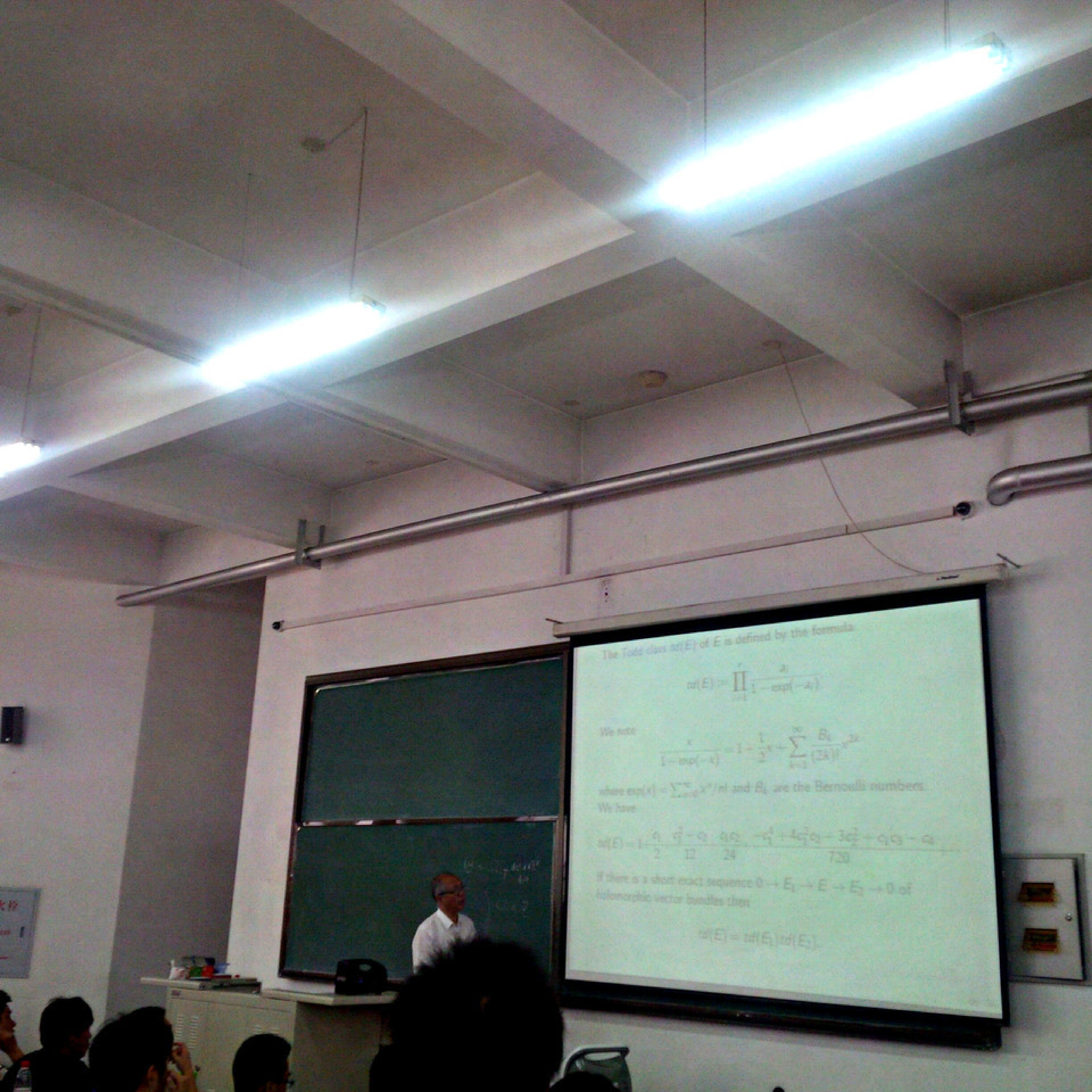
set: $k \leftarrow 0$, $\mathbf{w}_0 \leftarrow \mathbf{W}\mathbf{s}^0$, $t_0 \leftarrow 0$;
repeat
$\mathbf{w}^{k+1} \leftarrow \mathrm{prox}_{\Phi}\left(\mathbf{W}^T\left(\mathbf{s}^k + \frac{1}{L}(\mathbf{a} - \mathbf{A}\mathbf{s}^k)\right); \frac{\tau}{L}\right)$ (ISTA step)
$t_{k+1} \leftarrow \frac{1}{2}\left(1 + \sqrt{1 + 4t_k^2}\right)$
$\mathbf{v}^{k+1} \leftarrow \mathbf{w}^{k+1} + \frac{t_k - 1}{t_{k+1}}(\mathbf{w}^{k+1} - \mathbf{w}^k)$
$\mathbf{s}^{k+1} \leftarrow \mathbf{W}\mathbf{v}^{k+1}$
$k \leftarrow k + 1$
until stopping criterion
return $\mathbf{s}^k$


The Todd class $td(E)$ of E is defined by the formula
$$td(E) := \prod_{i=1}^{r} \frac{a_i}{1-\exp(-a_i)}.$$
We note
$$\frac{x}{1-\exp(-x)} = 1 + \frac{1}{2}x + \sum_{k=1}^{\infty} \frac{B_k}{(2k)!} x^{2k}$$
where $\exp(x) = \sum_{n=0}^{\infty} x^n/n!$ and B_k are the Bernoulli numbers.
We have
$$td(E) = 1 + \frac{c_1}{2} + \frac{c_1^2 + c_2}{12} + \frac{c_1 c_2}{24} + \frac{-c_1^4 + 4c_1^2 c_2 + 3c_2^2 + c_1 c_3 - c_4}{720} + \cdots$$
If there is a short exact sequence $0 \to E_1 \to E \to E_2 \to 0$ of holomorphic vector bundles then
$$td(E) = td(E_1)\,td(E_2).$$



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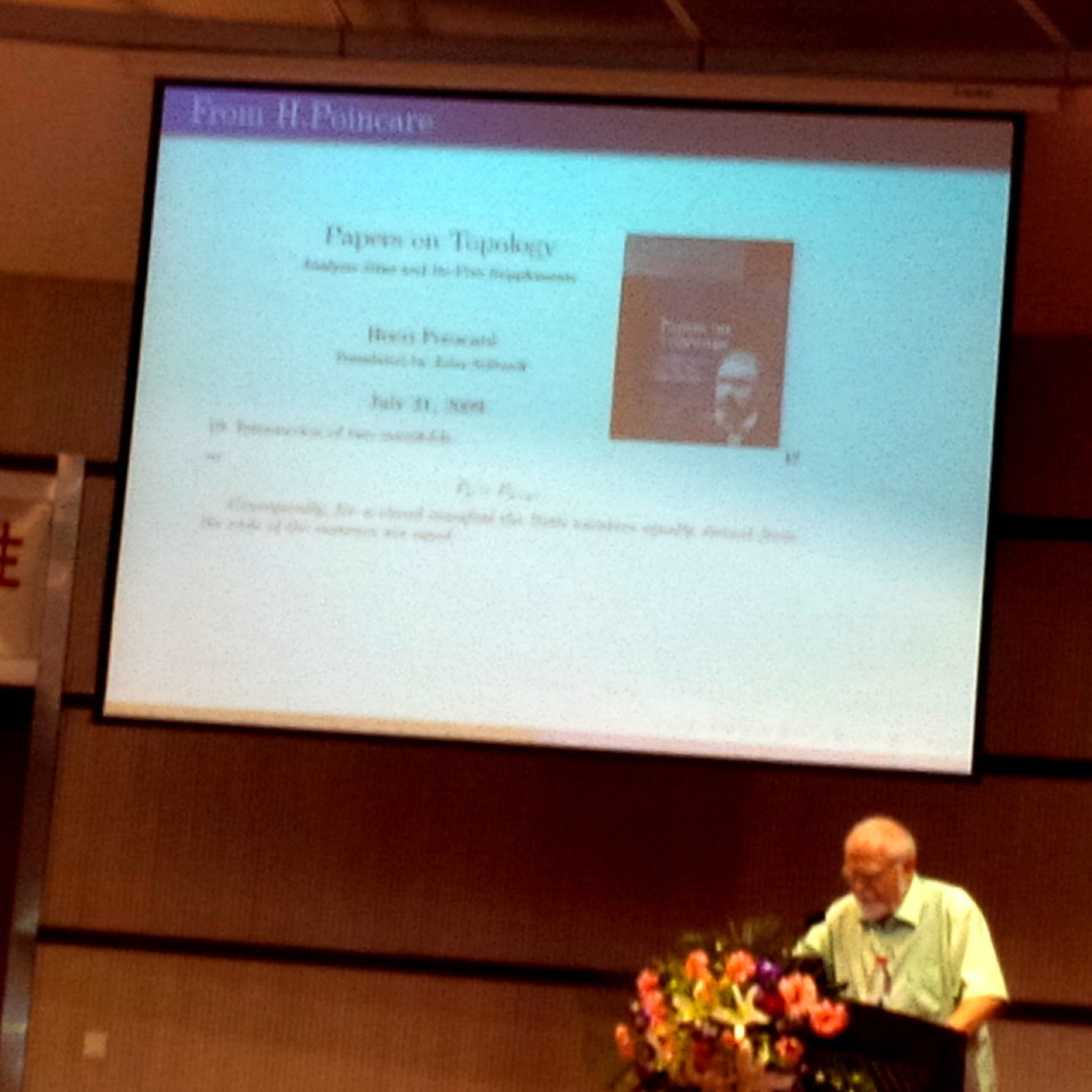
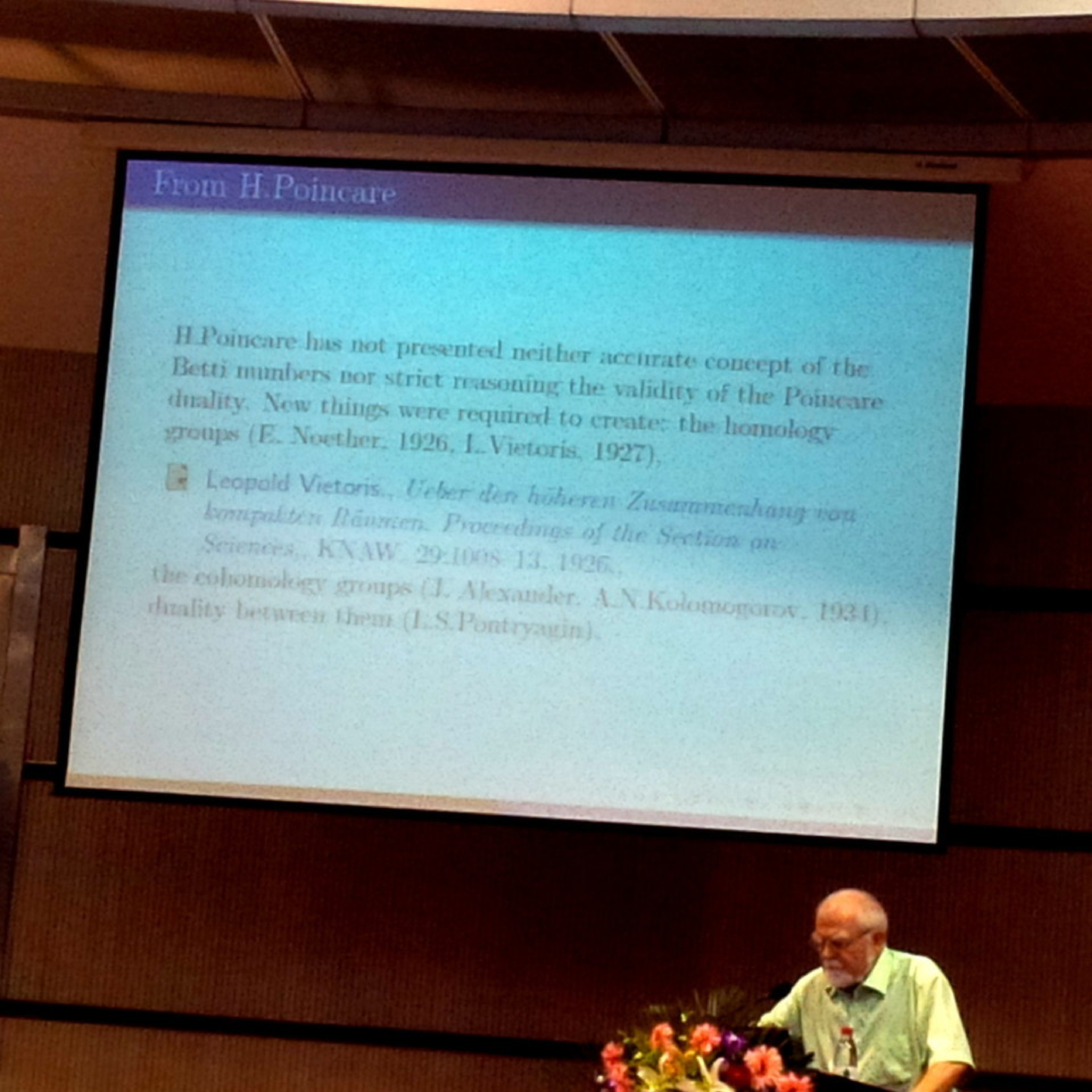
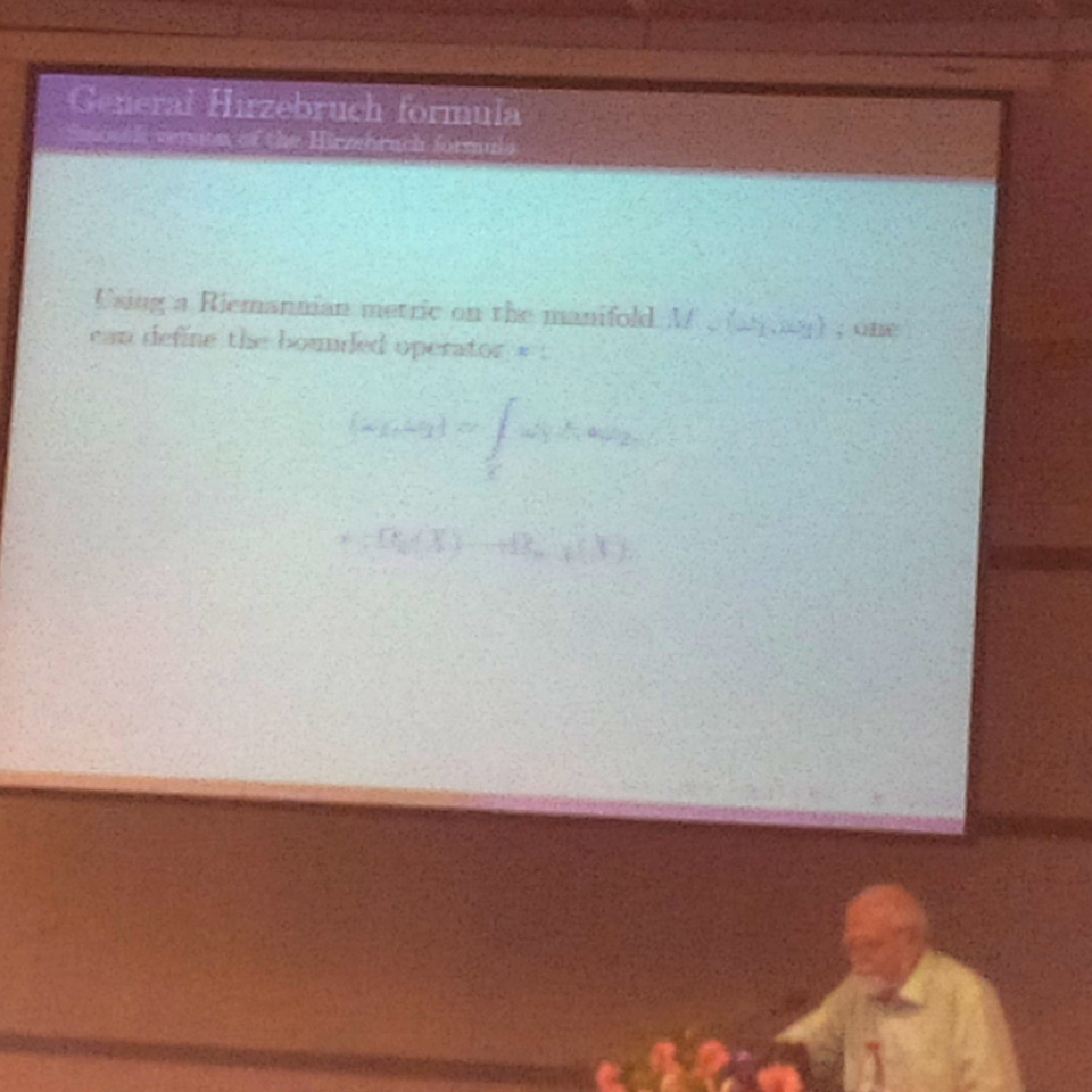
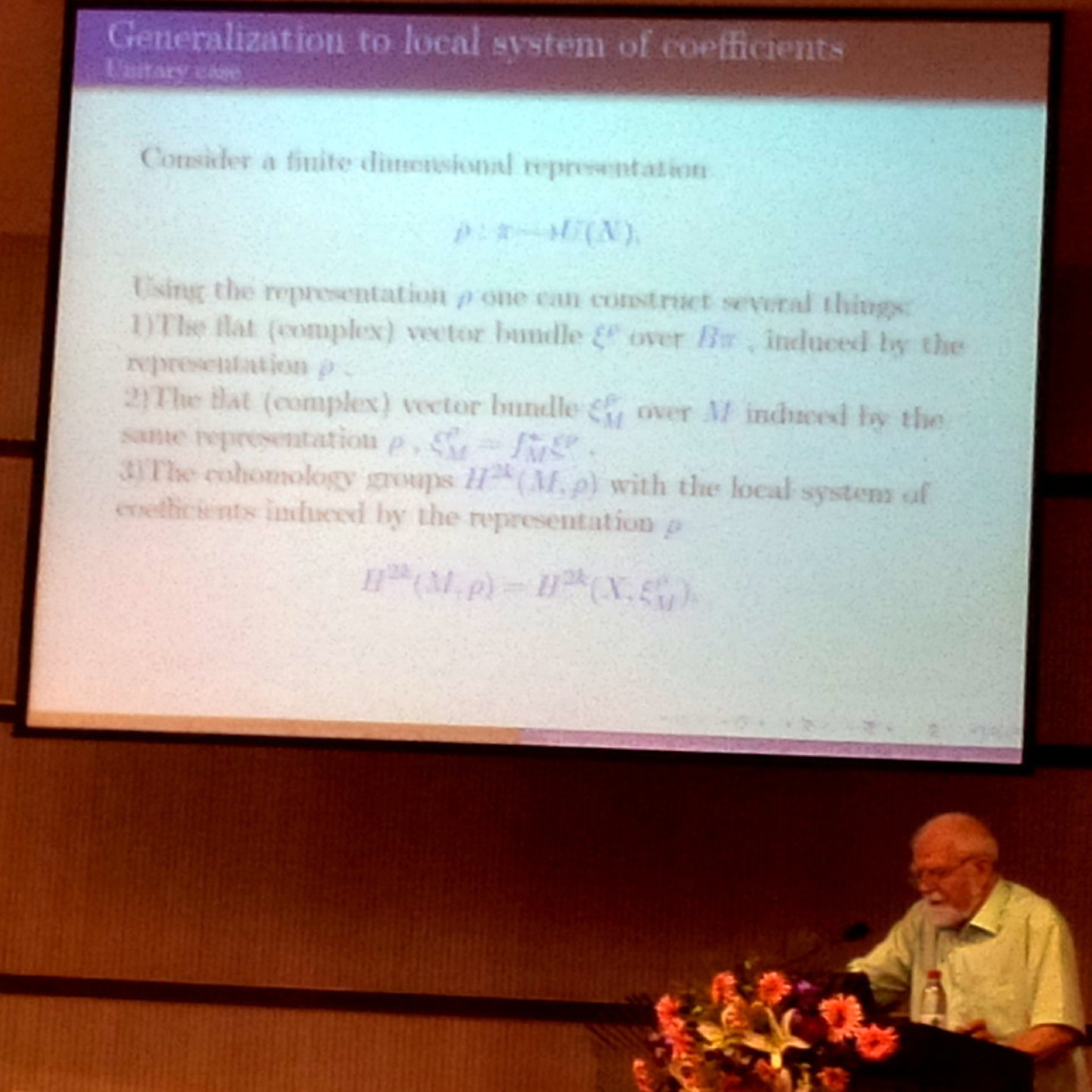
Milestones:

- 1930-s J. von Neumann: Mathematical foundations for quantum mechanics (series of papers "Rings of Operators" with F. Murray)
- 1943 I. Gelfand and M. Naimark: definition of a C^*-algebra
- 1970-s Results on Novikov conjecture (A. Mishchenko, G. Kasparov) and on C^*-algebra extensions (L. Brown, R. Douglas, P. Fillmore).
- 1990. Connes' book "Noncommutative Geometry"


From H.Poincare
Papers on Topology


From H.Poincare
H.Poincare has not presented neither accurate concept of the Betti numbers nor strict reasoning the validity of the Poincare duality. New things were required to create: the homology groups (E. Noether, 1926, L.Vietoris, 1927),
Leopold Vietoris., Ueber den höheren Zusammenhang von kompakten Räumen. Proceedings of the Section on Sciences., KNAW, 29:1008–13, 1926.
the cohomology groups (J. Alexander, A.N.Kolomogorov, 1934), duality between them (L.S.Pontryagin).


General Hirzebruch formula
version of the Hirzebruch formula
Using a Riemannian metric on the manifold
can define the bounded operator


Generalization to local system of coefficients
Unitary case
Consider a finite dimensional representation
$\rho : \pi \to U(N)$.
Using the representation ρ one can construct several things:
1)The flat (complex) vector bundle ξ^ρ over $B\pi$, induced by the representation ρ.
2)The flat (complex) vector bundle ξ^ρ_M over M induced by the same representation ρ, $\xi^\rho_M = f_M^* \xi^\rho$.
3)The cohomology groups $H^{2k}(M, \rho)$ with the local system of coefficients induced by the representation ρ
$H^{2k}(M, \rho) = H^{2k}(X, \xi^\rho_M)$.

Local system of coefficients

The elliptic operator

$$D = (d + d^*) : \Omega^+(M) \longrightarrow \Omega^-(M)$$

induces the operator

$$D_\rho = (D \otimes \xi^\rho) : \Omega^+(M, \xi^\rho) \longrightarrow \Omega^-(M, \xi^\rho).$$

So we have

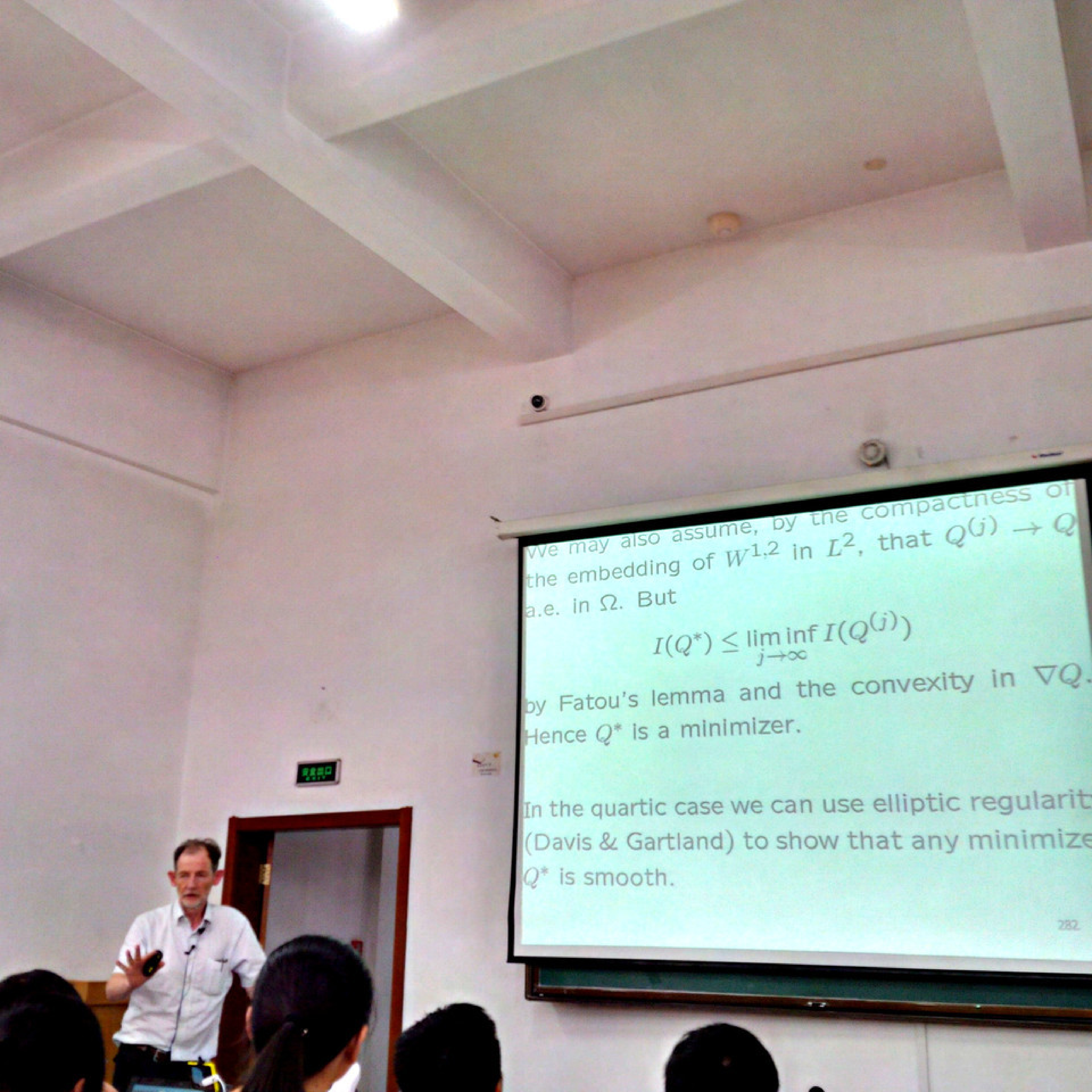
$$\textbf{sign}_{\,\rho}(M) = \textbf{index}\,(D_\rho).$$

Using Atiyah-Singer formula

$$\textbf{index}\,(D \otimes \xi) = 2^{2k}\,\langle L(M)\mathrm{ch}\xi, [M]\rangle,$$

for any vector bundle ξ over the manifold M we obtain

$$\textbf{sign}_{\,\rho}(M) = 2^{2k}\,\langle L(M)\mathrm{ch}\xi^\rho, [M]\rangle.$$


We may also assume, by the compactness of
the embedding of $W^{1,2}$ in L^2, that $Q^{(j)} \to Q$
a.e. in Ω. But
$I(Q^*) \le \liminf_{j\to\infty} I(Q^{(j)})$
by Fatou's lemma and the convexity in ∇Q.
Hence Q^* is a minimizer.
In the quartic case we can use elliptic regularit
(Davis & Gartland) to show that any minimize
Q^* is smooth.
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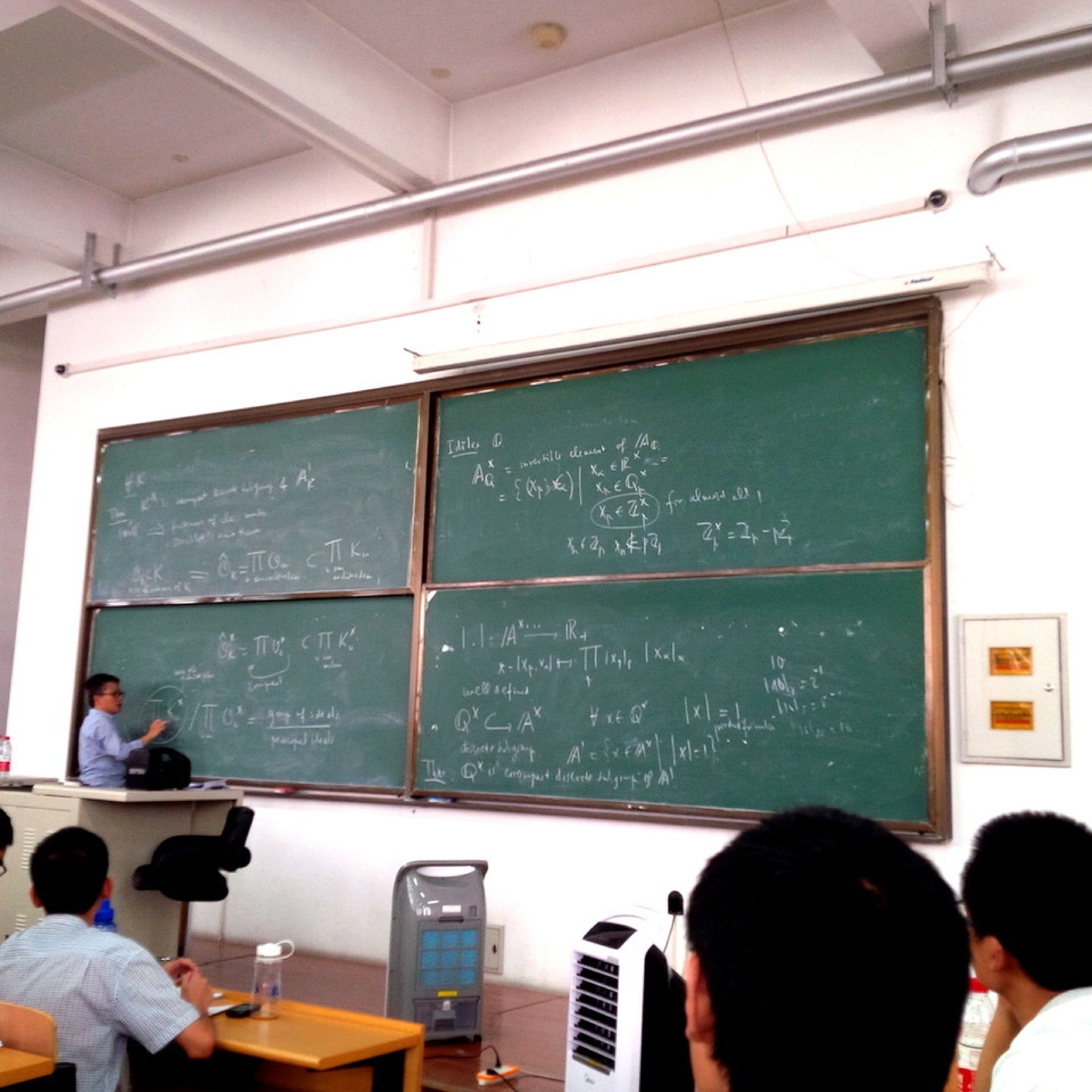

Idèles Q
= invertible element of A_Q
for almost all p
well defined
group of ideals
principal ideals
product formula
discrete subgroup
Thm: Q^× is cocompact discrete subgroup of A^1


Reduction modulo p
For all but finitely many primes p, E can be reduced to an elliptic curve E_p defined over $\mathbb{F}_p$. Those primes are said to be unramified with respect to E and p.
The Frobenius element $\mathrm{Fr}_p \in \mathrm{Gal}(\bar{\mathbb{F}}_p/\mathbb{F}_p)$ is defined by $\mathrm{Fr}_p(\alpha) = \alpha^{-p}$. There are not an unique way to lift Fr_p to an element of $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$. However,
for unramified p, $\rho_E(\mathrm{Fr}_p)$ is a well defined conjugacy class in $\mathrm{GL}_2(\mathbb{Q}_l)$.


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Langlands' reciprocity conjecture
Ngô Bảo Châu
G is assumed to be split for simplicity.
L_{F_v} is an extension of the local Weil groups W_{F_v} by a compact Lie group.
Admissible representation π_v are parametrized by homomorphisms $\phi_v : L_{F_v} \to \hat{G}$ extending the known parametrization of unramified representations.
For global field, there should be an extension L_F of the global Weil group W_F with homomorphism $L_{F_v} \to L_F$ also well defined up to conjugacy.
Automorphic representations π should be parametrized by homomorphisms $\phi : L_F \to \hat{G}$. Local parameters $\phi_v : L_{F_v} \to \hat{G}$ are obtained by restricting ϕ from L_F to L_{F_v}.


Langlands' functoriality conjecture
Let $\rho : \hat{H} \to \hat{G}$ be a homomorphism. For every automorphic representation $\pi_H = \bigotimes_v \pi_{H,v}$ of H, there exists an automorphic representation $\pi = \bigotimes_v \pi_v$ of G such that if at an unramified place v, $\pi_{H,v}$ is parametrized by a semisimple conjugacy class $s_v \in \hat{H}$, then π_v is also unramified and parametrized by the conjugacy class $\rho(s_v)$.


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自由
友善


哈尔滨啤酒
冰纯
一起哈啤


in the set of
$\in SL_2(\mathbb{Q})$
class
such that